Thermal Effects on Weak Waves in a Radiative Magnetogasdynamic Media

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The singular surface theory has been used to determine the behavior of weak waves under the combined influence of time-dependent gasdynamic, radiation, and electromagnetic fields. The role of thermal radiation and conduction in the growth or decay of a wave has been studied under a quasiequilibrium and quasi-isotropic hypothesis of the differential approximation to the radiative heat-transfer equation. It is shown that there are two distinct modes of wave propagation, namely, a radiation-induced wave and a modified magnetogasdynamic wave. It is observed that a radiation-induced wave decays rapidly and has a negligibly small influence on the gasdynamic field under the nonrelativistic limit, whereas a magnetogasdynamic wave grows into a shock wave under nonlinear steepening effects. Two cases of diverging and converging waves are discussed to answer the question as to when a shock wave appears.

Introduction

OBTAINING analytical solutions to nonlinear problems involving a complex interaction of gasdynamic, radiation, and electromagnetic fields has been a subject of great interest to researchers. A complete analysis of high-temperature flowfields depends on the study of these interactions. The inclusion of radiation makes the governing gasdynamic relationships a complex set of nonlinear integrodifferential equations when accounting for the frequency dependency of the radiation field. Such a situation arises when a solar wind of fully ionized plasma interacts with a plasma column of Earth's atmosphere and compresses it. As a result of interaction, weak wave characteristics emerge and, with changing inclinations, they intersect to form a shock wave.[11] The study of this phenomenon of interactions is of vital importance to space scientists. This paper provides a theoretical base as to how and when a shock wave will be formed.

During the last two decades, many researchers such as Wang,[1] Helliwell,[2] Helliwell and Mosa,[3] Sharma and Shyam,[4] and Sharma and Mosa[5] have studied some problems of the interaction of radiation and gasdynamic fields. Ram[6] studied the local and global behavior of acceleration waves by taking into account the transparent approximation to the radiative heat-transfer equation. In these studies, the approximations were too strong and they dealt with only simple cases in which the gas is either optically thin or thick. Also in Ref. 6, the neglect of the time dependence of the radiation field suppresses one mode of wave propagation that is excited by the radiation. Thus, the exact behavior of waves in radiation gasdynamics was not fully understood. Ram and Pandey[7] also studied the behavior of weak discontinuities in radiating gases by neglecting the thermal conduction effects. However, it is more desirable to account for the contribution of the thermal conduction effects along with that of the radiation and electromagnetic fields in solving problems of gaseous flows under high-temperature conditions. Under steady-state conditions and other simplifying approximations of optically thick or thin limits, the scientists are deprived of the knowledge of a spontaneous occurrence of a radiation-induced disturbance. The purpose of this paper is to study the growth and decay properties of weak magnetogasdynamic (MGD) waves by taking into account the effects of thermal conduction and the time-dependent radiation field interacting with the magnetogasdynamics field.

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Basic Equations and Modes of Propagation

The set of nonlinear differential equations governing the three-dimensional unsteady flow for radiation magnetogasdynamics is

$$\frac{\partial \rho}{\partial t} + u_i \rho_{,i} + \rho u_{i,i} = 0 \tag{1}$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j u_{i,j} + p,i + p^R,i + \mu H_j (H_{j,i} - H_{i,j}) = 0 \tag{2}$$

$$\frac{\partial H_i}{\partial t} + u_j H_{i,j} - H_j u_{i,j} + H_i u_{j,j} = 0 \tag{3}$$

$$\frac{\partial E}{\partial t} + [u_i(E + p + p^R) + q^R_{,i} - KT_{,i}]_{,i} = 0 \tag{4}$$

$$p = \rho R T \tag{5}$$

where u_i, H_i, p, ρ, R, and T represent, respectively, the components of the flow velocity, components of the magnetic field, gas pressure, gas density, gas constant, and absolute gas temperature; E is the total energy per unit volume, p^R the radiation pressure, and q_i^R the radiative heat flux vector. The constants K and μ represent the coefficient of thermal conduction and the magnetic permeability while t denotes time. A comma followed by an index i denotes the partial differentiation with respect to the spatial coordinate x^i.

The equations of radiative heat transfer within the differential approximation[8] may be written as a pair of equations:

$$\frac{\partial E^R}{\partial t} + q^R_{i,i} + \alpha(CE^R - 4\sigma T^4) = 0 \tag{6}$$

$$\frac{1}{C}\frac{\partial q_i^R}{\partial t} + \frac{1}{3} CE^R_{,i} + \alpha q_i^R = 0 \tag{7}$$

where $E^R = 3p^R$ is the radiative energy density per unit volume, α a gray gas absorption coefficient, C the speed of light, and σ the Stefan-Boltzmann constant.

The energy equation (4) for a perfect-gas model can be expressed in the form,

$$\frac{\partial p}{\partial t}+u_i p,i+3(\gamma-1)\left(\frac{\partial p^R}{\partial t}+u_i p^R,i\right)$$
$$+u_{i,i}[\gamma p+4(\gamma-1)p^R]+4(\gamma-1)(q^R_{i,i}-KT_{,ii})$$
$$+(\gamma-1)\mu u_i H_j(H_{i,j}-H_{j,i})=0 \tag{8}$$

where γ denotes the ratio of specific heats.

The previous set of nonlinear partial differential equations is hyperbolic in nature and admits discontinuities in some of the flow properties. We aim to study a surface of discontinuity in the flowfield across which the flow parameters p, ρ, u_i, H_i, q^R_i, p^R, etc., are essentially continuous, but finite discontinuities in their derivatives are permitted. Such a jump discontinuity is defined as a weak wave. The first-order geometric and kinematic compatibility conditions due to Thomas[9] are

$$[Z_{,i}]=Bn_i, \qquad \left[\frac{\partial Z}{\partial t}\right]=-BG \tag{9}$$

where Z may represent any of the flow variables and the scalar function $B=[Z_{,i}]n_i$ is defined over the surface of discontinuity, G is the normal speed of propagation of the moving wave front, and n_i unit normal components of the wave front.

From the law of conservation of energy across a weak discontinuity surface, for which $K\neq 0$, we have from Eq. (4),

$$K[T_{,i}]n_i=[q^R_i]n_i=0 \tag{10}$$

Differentiating Eq. (5) with respect to x_i and then applying the compatibility conditions of Eq. (9) and using Eq. (10), we get

$$\rho_0\xi=p_0\zeta \tag{11}$$

where $\xi=[p,i]n_i$, $\zeta=[\rho_{,i}]n_i$, and the suffix 0 denotes the evaluation just ahead of the propagating wave surface.

Evaluating the Eqs. (1-3), (6), and (7) on the wave front, we get

$$(u_{0n}-G)\zeta+\rho_0\lambda_i n_i=0 \tag{12}$$

$$\rho_0(u_{0n}-G)\lambda_i+\xi n_i+\theta n_i+\mu H_{0j}\eta_j n_i=0 \tag{13}$$

$$(u_{0n}-G)\eta_i+H_{0i}\lambda_j n_j=0 \tag{14}$$

$$-3G\theta+\epsilon_i n_i=0 \tag{15}$$

$$-G\epsilon_i+C^2\theta n_i=0 \tag{16}$$

where $\theta=[p^R_{,i}]n_i$, $\epsilon_i=[q^R_{i,j}]n_j$, $\lambda_i=[u_{i,j}]n_j$, and $u_{0n}=u_{0i}n_i$.

Equations (11-16) form a determinate set of nine homogeneous equations for nine unknowns λ_i, ϵ_i, θ, ξ, and ζ. The system has a nontrivial solution if $G=u_{0n}+C_{\text{eff}}$ or $G=C/\sqrt{3}$, where $C_{\text{eff}}=(a_0^2+b_0^2)^{1/2}$. The quantities a_0 and b_0 are, respectively, the isothermal speed of sound and Alfv́an speed as given by $a_0^2=(p_0/\rho_0)$ and $b_0^2=\mu H_0^2/\rho_0$. This implies that the flowfield admits two modes of wave propagation, one propagating with the speed $G=u_{0n}+C_{\text{eff}}$ and the other with the speed $C/\sqrt{3}$.

Behavior of a Radiation-Induced Wave

For a radiation-induced wave, $G=C/\sqrt{3}$. Substituting for G in Eqs. (12), (13), (15), and (16) and using Eq. (11), we obtain

$$\lambda^R=\epsilon^R\left[3\rho_0 C^2\left(\frac{1}{3}-\frac{C^2_{\text{eff}}}{C^2}\right)\right]^{-1} \tag{17}$$

$$\zeta^R=\epsilon^R\left[\sqrt{3}C^3\left(\frac{1}{3}-\frac{C^2_{\text{eff}}}{C^2}\right)\right]^{-1} \tag{18}$$

$$\xi^R=\epsilon^R a_0^2\left[\sqrt{3}C^3\left(\frac{1}{3}-\frac{C^2_{\text{eff}}}{C^2}\right)\right]^{-1} \tag{19}$$

$$\theta^R=\epsilon^R/C\sqrt{3} \tag{20}$$

where $\lambda^R=\lambda^R_i n_i$, $\epsilon^R=\epsilon^R_i n_i$, and superscript R denotes a jump discontinuity associated with the radiation-induced wave. To determine these jump discontinuities, we still need to determine ϵ^R. Partially differentiating Eqs. (6) and (7) with respect to x^i and t and using the compatibility conditions [Eq. (9)], we get a differential equation of the form,

$$\frac{\delta\epsilon^R}{\delta t}+C\left(\alpha-\frac{1}{\sqrt{3}}\,\Omega\right)\epsilon^R=0 \tag{21}$$

Equation (21) governs the growth or decay behavior of the amplitude of a radiation-induced weak wave. The mean curvature $\Omega(t)$ at any point of the wave surface has a representation of the form,[9]

$$\Omega=\frac{\Omega_0-K_0Gt}{1-2\Omega_0Gt+K_0G^2t^2} \tag{22}$$

where $\Omega_0=(K_1+K_2)/2$ and $K_0=K_1K_2$ are the mean and Gaussian curvatures of the wave surface, respectively, at $t=0$ with K_1 and K_2 being the principal curvatures and G the constant speed of propagation of wave. Since a radiation-induced wave is divergent, both K_1 and K_2 are negative. Solving Eq. (21), we get

$$\epsilon^R=\epsilon^R_0 I\exp(-\alpha Ct) \tag{23}$$

where

$$I=[(1-3^{-1/2}K_1Ct)(1-3^{-1/2}K_2Ct)]^{-1}$$

and ϵ^R_0 is the value of ϵ^R at $t=0$.

Since α and C are positive constants, it is evident from Eq. (23) that the amplitude ϵ^R of a radiation-induced wave decays rapidly and tends to zero as $t\to\infty$. Since C is very large, it follows from Eqs. (17-19) that any disturbance caused by a radiation-induced wave has a negligibly small influence on the nonrelativistic gasdynamic field.

Behavior of a Magnetogasdynamic Weak Wave

We shall now study the propagation of a weak magnetogasdynamic discontinuity into a medium that is in a constant state at rest. The jump discontinuities ξ, ζ, λ, and η_i are connected by relations

$$\lambda=C_{\text{eff}}\frac{\xi}{\bar{\rho}_0a_0^2}=C_{\text{eff}}\frac{\zeta}{\rho_0}=C_{\text{eff}}\eta_i\frac{H_{0i}}{H_0^2} \tag{24}$$

where $\eta_i=[H_{i,j}]n_j$ and $\lambda=[u_{i,j}]n_j\,n_i$ may be defined as the amplitude of the propagating discontinuity.

Since $u_{0n}=0$ and $G=C_{\text{eff}}$, it follows from Eqs. (13), (14), (16), and (24) that

$$\theta=0, \qquad \epsilon_i n_i=0 \tag{25}$$

If we partially differentiate Eqs. (1-3) with respect to x^k and evaluate them on the wave front by using the second-order compatibility conditions of Thomas,[9] we obtain a set of following equations:

$$-G\bar{\xi}+\frac{\delta\lambda}{\delta t}+\rho_0\bar{\lambda}_i n_i-2\rho_0\lambda\Omega+2\lambda\zeta=0 \tag{26}$$

$$-\rho_0 G\bar{\lambda}_i n_i+\bar{\xi}+\bar{\theta}+\mu H_{0j}\bar{\eta}_j+\rho_0\lambda^2-G\lambda\zeta+\mu\eta_j^2$$
$$+\rho_0\frac{\delta\lambda}{\delta t}-\mu\eta_j\eta_i n_i n_j=0 \tag{27}$$

$$-GH_{0i}\bar{\eta}_i+H_0^2 n_i\bar{\lambda}_i+\left[\frac{\delta\eta_i}{\delta t}+2\lambda\eta_i-\lambda_i\eta_j n_j\right]H_{0i}-2H_0^2\lambda\Omega=0 \tag{28}$$

where

$$\bar{\lambda}_i=[u_{i,jk}]n_j n_k,\quad \bar{\zeta}=[\rho_{,jk}]n_j n_k,\quad \bar{\xi}=[p,jk]n_j n_k$$
$$\bar{\theta}=[p^R_{,jk}]n_j n_k,\quad \bar{\eta}_i=[H_{i,jk}]n_j n_k$$

Partially differentiating Eq. (5) twice with respect to x^i and then evaluating on the wave front and using the second-order compatibility conditions,[9] we obtain a relation of the form,

$$\bar{\xi}-a_0^2\bar{\zeta}=\rho_0 R\bar{\phi} \tag{29}$$

where $\bar{\phi}=[T_{,ij}]n_i n_j$.

Evaluating Eq. (8) on the wave front, we get,

$$-G\bar{\xi}+[\gamma p_0+4(\gamma-1)p_0^R]\lambda-\frac{(\gamma-1)K}{\rho_0 R}(\bar{\xi}-a_0^2\bar{\zeta})=0 \tag{30}$$

Partially differentiating Eqs. (6) and (7) with respect to x^k and then evaluating on the wave front, we get

$$\bar{\theta}=0$$

Eliminating $\bar{\lambda}_i n_i$, $\bar{\xi}$, $\bar{\zeta}$, $\bar{\theta}$, and $\bar{\eta}_i$ from Eqs. (25–28) and using Eq. (24), we get

$$\frac{\delta\lambda}{\delta t}+\left[\frac{1}{2}\frac{\rho_0 R(1+4R_{p0})}{K}-C_{\text{eff}}\Omega\right]\lambda+\frac{(2a_0^2+b_0^2)}{2C_{\text{eff}}^2}\lambda^2=0 \tag{31}$$

where Ω is the mean curvature of the propagating wave surface.

Equation (31) can be rewritten as

$$\frac{\delta\lambda}{\delta t}+(\beta_0-C_{\text{eff}}\Omega)\lambda+\Gamma_0\lambda^2=0 \tag{32}$$

where

$$\beta_0=\frac{1}{2}\frac{\rho_0 R(1+4R_p)}{K}>0 \text{ and } \Gamma_0=\frac{2a_0^2+b_0^2}{2C_{\text{eff}}^2}>0$$

Equation (32) is the Bernoulli-type differential equation governing the growth and decay behavior of a magnetogasdynamic weak wave in a radiation-induced flowfield.

The mean curvature Ω of the wave surface propagating normal to itself into a uniform medium at rest can be expressed in the form,[9]

$$2\Omega=\frac{K_1}{1-K_1C_{\text{eff}}t}+\frac{K_2}{1-K_2C_{\text{eff}}t}$$

where K_1 and K_2 are the principal curvatures of the initial wave surface at time $t=0$. Integrating Eq. (32), we get

$$\lambda=\lambda_0[\exp(-\beta_0 t)(1-K_1C_{\text{eff}}t)^{-1/2}(1-K_2C_{\text{eff}}t)^{-1/2}]$$
$$\times\left[1+\lambda_0\Gamma_0\int_0^t\exp(-\beta_0 t')(1-K_1C_{\text{eff}}t')^{-1/2}\right.$$
$$\left.\times(1-K_2C_{\text{eff}}t')^{-1/2}\text{d}t'\right]^{-1} \tag{33}$$

where λ_0 is the initial amplitude at $t=0$.

To study the physical aspects of the wave amplitude, we shall discuss the following two cases of diverging and converging waves.

Case I: Diverging Waves

For diverging waves, both the initial principal curvatures K_1 and K_2 are negative and we may suppose, without any loss of generality, that $|K_1|\geqq|K_2|$, so that Eq. (33) can be expressed in the form

$$\lambda(t)=\lambda_0\exp(-\beta_0 t)\left[\frac{\phi_1(t)}{1+\lambda_0\Gamma_0\phi_2(t)}\right] \tag{34}$$

where

$$\phi_1(t)=[(1+|K_1|C_{\text{eff}}t)^{-1/2}(1+|K_2|C_{\text{eff}}t)^{-1/2}]$$

$$\phi_2(t)=\int_0^t\phi_1(t')\exp(-\beta_0 t')\text{d}t'$$

Since β_0 is a positive constant, the function $\phi_1(t)$ is nonnegative and monotonically decreasing and tends to zero as $t\to\infty$. If $\lambda_0>0$, then $\lambda(t)$ is also non-negative and $\lim_{t\to\infty}|\lambda(t)|=0$. This situation arises in the case of expansion waves that ultimately decay in time and damp out.

If $\lambda_0<0$, then $\phi_1(t)$ is positive, continuous, and an monotonically decreasing function. Furthermore, there exists a critical value λ_c of $|\lambda(t)|$ given by

$$\lambda_c=[\Gamma_0\phi_2(\infty)]^{-1}>0,\text{ for }\beta_0\neq 0 \tag{35}$$

If $|\lambda_0|<\lambda_c$, then there exists a damping compression wave such that $\lim_{t\to\infty}|\lambda(t)|=0$. If $|\lambda_0|=\lambda_c$, then $\lim_{t\to\infty}|\lambda(t)|=(\beta_0/\Gamma_0)$, which shows that the wave will neither terminate into a shock wave nor damp out. It ultimately takes a stable wave form. Further, if $|\lambda_0|>\lambda_c$, then there exists a critical time $t_c>0$ such that

$$\phi_2(t_c)=\frac{1}{|\lambda_0|\Gamma_0} \tag{36}$$

and $\lim_{t\to t_c}|\lambda(t)|=\infty$. This situation arises for compressive waves that grow in time and terminate into shock waves within a finite time $t_c>0$ due to nonlinear steepening effects. From Eqs. (35) and (36), we have

$$\frac{\text{d}\lambda_c}{\text{d}\beta_0}=\Gamma_0\lambda_c^2\int_0^\infty t\phi_1(t)\text{d}t>0 \tag{37}$$

$$\frac{\text{d}t_c}{\text{d}\beta_0}=\frac{\exp(\beta_0 t_c)}{\phi_1(t_c)}\int_0^{t_c}t\exp(-\beta_0 t)\phi_1(t)\text{d}t>0 \tag{38}$$

From Eqs. (37) and (38), it follows that both λ_c and t_c increases with the increase of β_0. As a consequence, we may conclude that the steepening of the waves is resisted by radiation effects and the breaking of waves into shock waves is either disallowed or delayed. On the other hand, the thermal conduction effects increase the steepening tendency and accelerate the process of breaking of a wave into a shock wave. This exhibits an interesting competition in the sense that the thermal radiation has a stabilizing tendency, while the thermal conduction has a destabilizing tendency.

Also from Eqs. (35) and (36), we obtain

$$\frac{\mathrm{d}\lambda_c}{\mathrm{d}\Gamma_0} = -\frac{1}{\Gamma_0^2 \phi_2(\infty)} < 0 \tag{39}$$

$$\frac{\mathrm{d}t_c}{\mathrm{d}\Gamma_0} = -\frac{e^{\beta_0 t_c}}{\Gamma_0^2 |\lambda_0| \phi_1(t_c)} < 0 \tag{40}$$

Since Γ_0 decreases with the increase of magnetic field effects, the inequalities in Eqs. (39) and (40) show that both λ_c and t_c increase under magnetic field effects. This implies that the presence of electromagnetic field in the flowfield has a stabilizing effect and delays the breaking of the waves.

To study the growth or decay behavior of compressive waves ($\lambda_0 < 0$), we introduce the following dimensionless parameters:

$$\lambda = \lambda^* \delta, \quad \eta = \frac{t - t^*}{t^*} \quad \text{or} \quad t = t^*(\eta + 1) \tag{41}$$

where λ^* and t^* are the initial values of λ and t.

After introducing Eq. (41), Eq. (32) assumes the form

$$\frac{\mathrm{d}\delta}{\mathrm{d}\eta} + (\beta - C_{\mathrm{eff}} t^* \Omega)\delta + \Gamma \delta^2 = 0 \tag{42}$$

where

$$\beta = \frac{1}{2K_p}(1 + 4R_p), \quad K_p = \frac{K}{p_0 R t^*}$$

$$\Gamma = \left[\frac{2 + b_0^2/a_0^2}{2(1 + b_0^2/a_0^2)}\right] \lambda^* t^*$$

Case II: Converging Waves

For converging waves, at least one of the principal curvatures (K_1 or K_2) is positive. If K_1 is positive and K_2 negative or if both are positive and $K_1 > K_2$, then we can define $t^* = (K_1 C_{\mathrm{eff}})^{-1}$. If the propagating wave is an expansion wave, then $\lambda_0 > 0$ and Eq. (34) is of the form

$$\lambda(t) = \lambda_0 F(t) \Big/ \left[1 + \lambda_0 \Gamma_0 \int_0^t F(\tau)\mathrm{d}\tau\right]$$

where

$$F(t) = e^{-\beta_0 t}(1 - K_1 C_{\mathrm{eff}} t)^{-1/2}(1 - K_2 C_{\mathrm{eff}} t)^{-1/2}$$

Since $F(t)$ is positive and an integrable function in every finite subinterval of $0 \leqq t \leqq t^*$, we have $\lim_{t \to t^*} F(t) = \infty$ and

$$\int_0^{t^*} F(\tau)\mathrm{d}\tau < \infty, \quad \text{for } K_1 \neq K_2$$

It follows that $\lim_{t \to t^*} \lambda(t) = \infty$

Also, if $K_1 = K_2 > 0$, then

$$\lim_{t \to t^*} \int_0^t F(\tau)\mathrm{d}\tau = \infty \quad \text{and} \quad \lim_{t \to t^*} \lambda(t) = \infty$$

This shows that all converging expansion waves will form a caustic at time t^*.[10]

If $\lambda_0 < 0$ in the case of compressive waves, then there exists a finite critical amplitude η_c given by

$$\eta_c = \left[\Gamma_0 \int_0^t F(t)\mathrm{d}t\right]^{-1}$$

such that when $|\lambda_0| < \eta_c$, then $\lim_{t \to t^*} |\lambda(t)| = \infty$. This shows that even a compressive wave with its initial amplitude numerically less than the critical one will form a caustic at t^* and the shock formation is disallowed. On the other hand, if $|\lambda_0| > \eta_c$, then a shock wave will be formed as a consequence of nonlinear steepening at a finite time $t_c < t^*$, where t_c is given by

$$\int_0^{t_c} F(t)\mathrm{d}t = \frac{1}{\Gamma_0 |\lambda_0|} \tag{43}$$

If $|\lambda_0| = \eta_c$, then we have

$$\lim_{t \to t_c} |\lambda(t)| = \infty \quad \text{and} \quad t_c = t^*$$

This shows that under this case there occurs a simultaneous appearance of a shock wave and a caustic at times $t = t^*$.

INDICATIONS

II $R_p = 0{\cdot}5$, $K_p = \frac{K}{P_0 R t^*} = 1$, $b_H = \frac{b_0}{a_0} = 0{\cdot}5$, $\nu^* = \lambda^* t^* = -2$, $\nu_c = 5/3 = \frac{\beta_0}{\Gamma_0}$.

III $R_p = 1$, $K_p = 1$, $b_H = 0{\cdot}5$, $\nu^* = -1$, $\nu_c = 25/9$.

IV $R_p = 0{\cdot}5$, $K_p = 2$, $b_H = 0{\cdot}5$, $\nu^* = -1$, $\nu_c = 5/6$.

VIII $R_p = 0{\cdot}5$, $K_p = 3$, $b_H = {\cdot}5$, $\nu^* = -2$, $\nu_c = {\cdot}555$.

IX $R_p = 1$, $K_p = 2$, $b_H = {\cdot}5$, $\nu^* = -1$, $\nu_c = 1{\cdot}388$.

X $R_p = 1$, $K_p = 3$, $b_H = {\cdot}5$, $\nu^* = -2$, $\nu_c = {\cdot}9259$.

XI $R_p = 1$, $K_p = 3$, $b_H = {\cdot}5$, $\nu^* = -1$, $\nu = 25/27$.

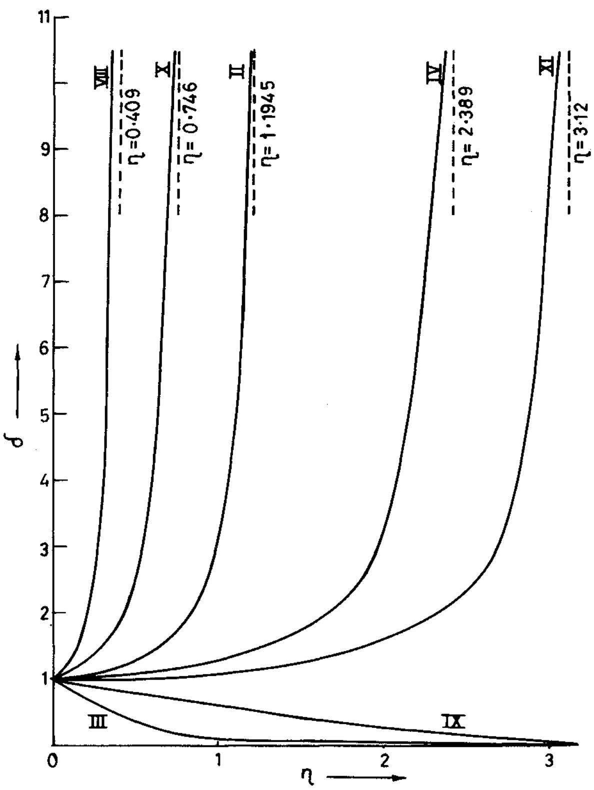

Fig. 1 Role of thermal conduction and radiation pressure in the growth and decay behavior of compressive waves.

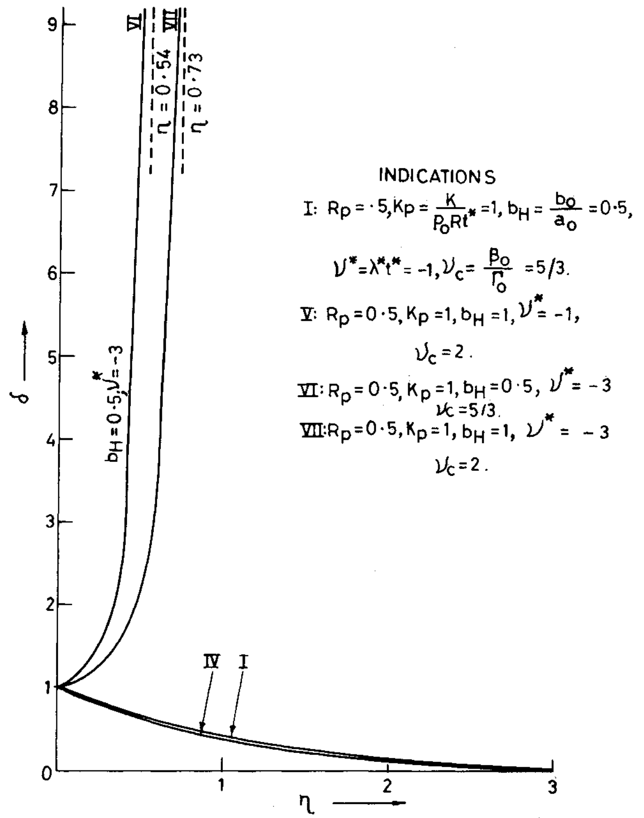

Fig. 2 Effect of magnetic field in the growth and decay of compressive waves.

To study curvature effects on diverging waves, we differentiate Eq. (36) with respect to $|K_1|$ to get

$$\frac{\mathrm{d}t_c}{\mathrm{d}\,|K_1|}=\frac{C_{\mathrm{eff}}}{2\phi_2(t_c)}\int_0^{t_c}\frac{t\,\phi_2(t)}{(1+|K_1|C_{\mathrm{eff}}t)}\,\mathrm{d}t>0 \tag{44}$$

which shows that the critical time t_c increases under curvature effects. If we partially differentiate Eq. (43) with respect to $K_1>0$, we get

$$\frac{\mathrm{d}t_c}{\mathrm{d}K_1}=-\frac{C_{\mathrm{eff}}}{2F(t_c)}\int_0^{t_c}\frac{tF(t)}{(1-K_1C_{\mathrm{eff}}t)}\,\mathrm{d}t<0 \tag{45}$$

The inequality of Eq. (45) implies that for converging waves the critical time t_c decreases under curvature effects. Thus, we conclude that under curvature effects the stability of diverging wave increases while that of converging wave decreases.

Conclusions

It has been shown that an unsteady radiation field gives rise to a radiation-induced wave that has a negligibly small influence on the gasdynamic field. The growth or decay behavior of a radiative magnetogasdynamic weak discontinuity depends on the interplay by the radiation pressure, the thermal conduction, and the magnetic field. Figures 1 and 2 show that the tendency of the radiation pressure and magnetic field is to resist the shock formation, whereas thermal conduction has a destabilizing effect and accelerates the shock formation. The growth or decay of compressive waves depend on whether an initial wave strength $\nu^*=\lambda^*t^*$ is numerically below or above a certain critical value $\nu_c=\beta_0/\Gamma_0$, which is a function of various physical parameters.

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